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LIST PRICING AND PURE STRATEGY OUTCOMES IN A BERTRAND-EDGEWORTH DUOPOLY *

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Abstract

Non-existence of a pure strategy equilibrium in a Bertrand-Edgeworth duopoly model is analyzed. The standard model is modified to include a list pricing stage and a subsequent price discounting stage. Both firms first simultaneously choose a maximum list price and then decide to lower the price, or not, in a subsequent discounting stage. List pricing works as a credible commitment device that induces the pure strategy outcome. It is shown that for a general class of rationing rules there exists a sub-game perfect equilibrium that involves both firms playing pure strategies. This equilibrium payoff dominates any other sub-game perfect equilibrium of the game. Further unlike the dominant firm interpretation of a price leader, we show that the small firm may have incentives to commit to a low price and in this sense assume the role of a leader.

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1. Introduction

There has been a renewed interest in models where firms price subject to predetermined capacity constraints (also known as Bertrand-Edgeworth models).¹ The appeal of the Bertrand-Edgeworth specification is that firms actually set prices assuming a very simple technology that captures differences in firm size. Osborne and Pitchick (1986) and Allen and Hellwig (1986b) argue that these models are a natural starting point for a theory of firm behavior in oligopoly.

A common feature of Bertrand-Edgeworth models is the non-existence, in general, of a pure strategy equilibrium. One way of avoiding this non-existence problem is the mixed strategy solution concept,² however, mixed strategies are not considered by some as a satisfactory explanation of pricing behavior by firms. For example, Shubik and Levitan (1980) consider mixed strategies as an “interesting extension of the equilibrium that is somewhat hard to justify.” Dixon (1987) finds them “implausible” while Friedman (1988) finds it “doubtful that the decision makers in firms shoot dice as an aid to selecting output or price.”

The mixed strategy outcome is not particularly troublesome when the number of firms in the industry is large. Allen and Hellwig (1986a) and Vives (1986) show, under different assumptions on the rationing function, that as the number of firms in a Bertrand-Edgeworth model grows the mixed strategy equilibrium converges in distribution to the competitive equilibrium. In this sense, Allen and Hellwig (1986b), while considering the non-existence of a pure strategy equilibrium a “drawback of the Bertrand-Edgeworth specification,” argue that in the large numbers case randomization in prices is “in some sense unimportant” as firms will set prices close to the competitive price with very high probability. The competitive result is robust to a change in the equilibrium concept. Dixon (1987) and Borgers (1992) obtain convergence to the competitive equilibrium using the ϵ -equilibrium and iterated elimination of dominated strategies solution concepts, respectively.

When the number of firms in the industry is small, particularly in the paradigmatic case of a duopoly, the previous approximation result does not apply. In this case the

¹Deneckere and Kovenock(1992) use such a model to explain price leadership in international trade; BJORSTEN (1994) uses it to analyze the effects of Voluntary Export Restraints; Iwano and Rosenbaum (1991) and Staiger and Wolak (1992) use the Bertrand-Edgeworth specification to study the relationship between prices and demand fluctuations in a dynamic model. Sorgard (1996) uses it to model a game of entry in an industry with a dominant firm.

²Maskin (1986) proves existence of a mixed strategy equilibrium for very general specifications of the Bertrand-Edgeworth model.

alternatives to the mixed strategy solution have involved models that assume sequential timing of firm moves. This is the approach that is followed in Shubik and Levitan (1980), Deneckere and Kovenock (1992) (henceforth DK), and Canoy (1996).

The paper closest to ours is that of DK (1992). DK analyze a price leadership model in a duopolistic market where the firms choose the timing of their price announcements, maximizing total discounted profits. They show that under efficient rationing, and when capacities are in the range where the simultaneous move game yields a mixed-strategy solution, the high capacity firm becomes a price leader. In their game, prices, once announced, cannot be changed. Firm 1 announces its price at the beginning of an *even* index while firm 2 announce its price at the beginning of an *odd* index. Given that both firms cannot choose prices in the same index, the *timing* choice in this sense is exogenous in their game. It should be, however, noted that they do not impose Stackelberg leader-follower roles beforehand.

Our paper provides an alternative to the sequential timing hypothesis by analyzing a natural extension of a Bertrand-Edgeworth model for which pure strategy equilibrium always exists. We consider a Bertrand-Edgeworth duopoly model where prices are determined *simultaneously* in two stages. In the first stage, both firms announce list prices simultaneously. In the second stage firms may discount from these list prices. In this sense prices are ex post (downward) *flexible* in our model. Under quite general assumptions about the rationing mechanism we show that there exists a subgame perfect equilibrium in which both firms play pure strategies and that this equilibrium payoff dominates any other subgame perfect equilibrium.

The motivation behind this two stage pricing structure is taken from list pricing³. List pricing is a widely extended trading institution where firms post prices for some period of time. These prices can then later be discounted. Our model does not provide an alternative solution concept to the mixed strategy Nash equilibrium but it yields the prediction that randomization by firms is not equilibrium behavior and it does so with a straightforward extension of the classical model. Further, we generalize some of the results of the Bertrand-Edgeworth literature which were only known to hold for the classical one stage pricing game.

The intuition behind our result is simple. In a Bertrand-Edgeworth equilibrium a firm may set a price such that its rival obtains higher profits from selling to the residual demand (than from setting some undercutting price). This price gives the rival a monopoly on the residual demand. By committing to a low list price a firm signals to its rival that

³Posting a maximum list price and later offering discounts is common practice in most retail markets.

it can act as a monopolist on the residual demand in the subsequent discounting stage. In this sense the list pricing institution acts as a facilitating collusion device between the firms⁴. There are some examples that suggest the empirical relevance of this type of pricing behavior in concentrated industries with a single dominant firm (see for instance Sorgard (1995)).

The paper is structured as follows: In Section 2 we present the basic model of a price setting duopoly with capacity constraints and specify a general residual demand function. In Section 3 we define the Edgeworth Price. This is useful for characterizing the pure and mixed strategy equilibria that arise in the game. In Section 4 we analyze the pricing equilibria of our list-pricing game and compare it to the equilibrium of the single stage pricing game. In Section 5 we explore the relationship between list pricing and price leadership. Section 6 concludes.

2. Residual Demand in a Bertrand-Edgeworth Duopoly

The classical Bertrand-Edgeworth game involves two stages, in the first stage (the production stage) firms simultaneously set capacities and in the second stage they simultaneously decide upon prices. Once prices are announced market demand is distributed between the firms according to some specified rationing rule which represents underlying consumer behavior and is assumed to be either efficient, or proportional.⁵

Consider a market with 2 firms that produce a homogenous good. The firms in the market face capacity restrictions $0 < k_i \leq D(0)$ and have zero costs. Suppose that the aggregate market demand, $D(p)$, is continuous and results in a strictly concave revenue curve, $pD(p)$. Suppose that $D(p)$ is positive downward sloping and twice differentiable on $(0, p^0)$ and zero for $p \geq p^0 > 0$. Let $P(q)$ denote the inverse demand function. Associated with the demand function and firm capacity we can define a firm's monopoly price $p_i^M = \arg \max p \min(D(p), k_i)$.

Given a vector of prices $p \in \mathbb{R}_+^2$ set by the firms we now discuss what firm i sells in the market.

⁴In a different context Holt and Scheffman (1988) analyze list pricing as a facilitating practice device.

⁵Efficient rationing also referred to as "surplus maximizing" and is used in Levitan and Shubik (1972), Kreps and Scheinkman (1983), Vives (1986), and Deneckre and Kovenock (1992). Proportional rationing is used in Beckman(1965), Allen and Hellwig (1986a-b), Dasgupta and Maskin (1986), Davidson and Deneckre (1986) (this last paper also has some results for a general class of rationing functions) and Deneckre and Kovenock (1992).

$$q_i(p_i, p_j) = \begin{cases} \min[k_i, D(p_i)] & p_i < p_j \\ \min[k_i, \frac{k_i}{k_i + k_j} D(p)] & p_i = p_j = p \\ \min[k_i, R(p_i, p_j, k_j)] & p_i > p_j \end{cases}$$

Where, $R(p_i, p_j, k_j)$ represents a general residual (or contingent demand) function, and is defined only for $p_i \geq p_j$. The residual demand function is determined by how the rationing of excess demand is modeled.

The Bertrand-Edgeworth literature has used one of two specifications of residual demand: proportional or efficient. To understand how they work we will suppose that consumers have a unitary demand, that firm j undercuts firm i , $p_i > p_j$, and that firm j cannot meet all its demand, $D(p_j) > k_j$. The proportional (or Beckman) residual demand specification results from the hypothesis that each potential consumer of firm j has an equal probability of being served. The residual demand facing the high priced firm is then given by,

$$R_B(p_i, p_j, k_j) = \max(D(p_i)(1 - \frac{k_j}{D(p_j)}), 0)$$

The efficient, or surplus maximizing, residual demand specification assumes that low priced goods are allocated to consumers with the highest valuation for the good.⁶ Under this assumption the high priced firm has residual demand,

$$R_E(p_i, p_j, k_j) = \max(D(p_i) - k_j, 0)$$

Proportional and efficient rationing are but two of the many reasonable specifications of residual demand. For instance one may assume that a proportion $1 - \lambda (> 0)$ of the low priced firm's capacity is allocated randomly among potential buyers while the remaining capacity goes to unsatisfied high valuation consumers, this would result in residual demand for the high priced firm of,

$$R_\lambda(p_i, p_j, k_j) = \max(\lambda(D(p_i) - k_j) + (1 - \lambda)D(p_i)(1 - \frac{k_j}{D(p_j)}), 0).$$

This function belongs to a class of residual demand functions for which our results hold. This class can be characterized by imposing the following restrictions on function $R(p_i, p_j, k_j) : A \rightarrow \mathbb{R}_+$ where, $A = \{(p, p', k') \in \mathbb{R}^3 : p \geq p' \geq 0, k' \geq 0\}$

⁶It should be noted that although efficient rationing maximizes consumer surplus (for a particular capacity constrained firm), it does not maximize total consumer surplus. Given capacities and prices, if the high priced firm can meet all its residual demand proportional rationing leads to greater total consumer and total surplus than efficient rationing.

1. $R(p_i, p_j, k_j)$ is continuous.
2. When $R(p_i, p_j, k_j) > 0$, it is strictly decreasing in p_i .
3. When $R(p_i, p_j, k_j) > 0$ then $R(p_i, p_j, k_j)p_i$ is strictly concave in p_i .
4. $\max(0, D(p_i) - k_j) \leq R(p_i, p_j, k_j) \leq \max(0, \min(D(p_j) - k_j, D(p_i)))$.
5. When $R(p_i, p_j, k_j) > 0$ it is strictly decreasing in p_j .

Properties (1),(2) and (3) guarantee that the residual demand function inherits certain regularity properties from the demand function. In order to understand property (4) consider what happens as p_j gets arbitrarily close to p_i . In this case the number of consumers of the low priced firm with a reservation price below p_i becomes arbitrarily small and the residual demand function is $D(p_i) - k_j$. With respect to the left hand side simply note that the low priced firm may never sell more than k_j units of the good.⁷

Property (5) refers to the fact that if firm j (the low price firm) lowers its price, p_j , more consumers enter the market and this reduces the amount of firm j 's output that is allocated to high valuation consumers. This in turn increases residual demand for the high price firm i . Thus firm i 's profits will rise as firm j lowers its price. The effect of firm j lowering its price on profits of firm i plays an important role in our results.

Further, it must be noted that the efficient residual demand is not included in the class of rationing functions we consider since it violates property (5). On the other hand our results do hold for functions that approximate efficient residual demand (ver small λ) as $R_\lambda(p_i, p_j, k_j)$ verifies properties 1-5 for any $0 \leq \lambda < 1$.

3. The Lower Bound of the Edgeworth Cycle

In a Bertrand-Edgeworth equilibrium a firm i may set a price such that its rival obtains higher profits from selling to the residual demand than from setting an undercutting price. We refer to the highest of such prices as the Edgeworth price, p_i^E (it is the lower bound of the Edgeworth cycle). The Edgeworth Price will be very useful in order to characterize the equilibria that arise in the pricing subgames that we study. If firm i sets a price greater than firm j 's monopoly price (p_j^M) it will surely be undercut, i.e., $p_i^E \leq p_j^M$.

Let us denote the price firm i sets to serve the residual demand by $p^R(p_j, k_j)$ (given that firm j chooses p_j). Then,

⁷Properties (1), (2) and (4) are proposed by Davidson and Deneckre (1986) for a "reasonable rationing function."

$$p^R(p_j, k_j) = \arg \max_{x \in [p_j, p_0]} R(x, p_j, k_j)x.$$

If firm i sets a price, p_i , such that (the competitive price) $P(k_1 + k_2) \leq p_i < p_j^M$ then the maximum profits that firm j obtains by setting a price p' less than p_i are bounded above and arbitrarily close to $\min(k_j, D(p')p')$. On the other hand the maximum profits that firm j obtains from acting on the residual demand is given by $R(p^R(p_i, k_i), p_i, k_i)p^R(p_i, k_i)$. This leads to our next result.

Theorem 3.1. p_i^E can be characterized by the unique price p that verifies,

$$\min(k_j, D(p))p = R(p^R(p, k_i), p, k_i)p^R(p, k_i).$$

Proof: We first prove that there is a unique p that verifies the equation. For $p \in [P(k_1 + k_2), p_j^M]$ the left hand side of the equation is strictly increasing and continuous and that the right hand side is decreasing (property 5) and continuous (property 1 and the Maximum Theorem). We have then that if the two functions cross they cross only once. We now prove that the two functions actually cross on $[P(k_1 + k_2), p_j^M]$. First note that if firm i sets a price, $P(k_i)$, (it has capacity enough to serve demand) then firm j will have incentives to undercut this price,

$$P(k_i) \min(k_{-i}, k_i) \geq \max_{x \in [P(k_i), p_0]} R(x, P(k_i), k_i)x$$

This is true since the left hand is positive and the right hand side is zero by the property (4) of residual demand. On the other hand if firm i sets the competitive price firm j will have no incentives to undercut it,

$$P(k_1 + k_2)k_{-i} \leq \max_{x \in [P(k_1 + k_2), p_0]} R(x, P(k_1 + k_2), k_i)x$$

This is true since the left hand side evaluated at $p = P(k_1 + k_2)$ is equal to the right hand side (by property (4) of residual demand).

Let us now denote the unique price that verifies the equality by \hat{p} . If firm i sets a price, p_i , such that $p_j^M > p_i > \hat{p}$ firm j will have incentives to undercut this price. On the other hand if firm i sets a price of \hat{p} the profits firm j may gain from undercutting firm i (setting a price below \hat{p}) are strictly less than $\min(k_j, D(\hat{p}))\hat{p}$, and acting on the residual demand as a monopolist will give it profits of exactly $\min(k_j, D(\hat{p}))\hat{p}$.

□

For the sake of convenience we will index two firms such that firm 1 has a higher Edgeworth price than firm 2, $p_1^E \geq p_2^E$. Note that further assumptions on the residual demand function would be needed to determine which firm will have the highest Edgeworth price although it is straightforward to see that if the residual demand function is of the type R_λ then there is a direct relation between the Edgeworth price and firm capacity where $k_i \geq k_j$ implies $p_i^E \leq p_j^E$. That is, the low capacity firm has a higher Edgeworth price than the high capacity firm.

4. The List Pricing Game

In the classical Bertrand-Edgeworth duopoly model firms are assumed to set prices simultaneously. In our extension of the classical model the price setting process is modeled in two stages. In the first stage each firm $i \in \{1, 2\}$ sets a list price p_i^L , and in the second stage firms are allowed to offer a discount on the list price. Given the discounted price p_i^d ($\leq p_i^L$) consumers make their purchasing decisions according to $q_i(p_i^d, p_j^d)$. For simplicity we do not consider list prices greater than p_i^0 . We refer to this extended model as the list pricing game. This game reflects the list pricing institution, also referred to as posted offer, that is prevalent in many industries.

In this section we prove that there exists a subgame perfect equilibrium to the list pricing game that involves no mixed strategies on the equilibrium path and we characterize this equilibrium. Furthermore we prove that if there exists a subgame perfect equilibrium that yields an outcome that is different from the proposed equilibrium the former is payoff dominated by the latter. If any preplay communication exists between the players then it could be argued that this dominating equilibrium would be chosen, in this sense the equilibrium that we propose is a focal point of the list pricing game.

4.1. The Discounting Subgame

We first verify the existence of an equilibrium to each discounting subgame given any pair of price ceilings $p_i^L \geq 0$. The proof is a straightforward application of Theorem 5 in Dasgupta and Maskin (1986a).

Theorem 4.1. *The discounting subgame has a (mixed) Nash equilibrium for any (p_1^L, p_2^L) .*

Proof: Note that each firm's action space $[0, p_i^L]$ is a closed interval and the profit (payoff) function of each firm,

$$\pi_i(p_i, p_j) = p_i q_i(p_i, p_j)$$

is continuous except on a subset of

$$A^*(i) = \{(p_i, p_j) \in [0, p_i^L] \times [0, p_j^L] | p_i = p_j\}$$

By proving that i) $\pi_1(p_1, p_2) + \pi_2(p_2, p_1)$ is continuous and ii) that $\pi_i(p_1, p_2)$ is weakly lower semi-continuous we may apply Dasgupta and Maskin (1986a) to obtain the desired existence result.

- i) The only possible discontinuity of $\pi_1(p_1, p_2) + \pi_2(p_2, p_1)$ occurs in when $p_1 = p_2 \geq P(k_1 + k_2)$. Consider a series of prices $(p_{it}, p_{jt}) \rightarrow (p^*, p^*)$ we then have

$$D(\bar{p}_t) \leq q_i(p_{ij}, p_{jt}) + q_j(p_{jt}, p_{it}) \leq D(\underline{p}_t)$$

where $\bar{p}_t \in \max_i p_{it}$ and $\underline{p}_t \in \min_i p_{it}$. Now let $\epsilon(t) = \bar{p}_t - \underline{p}_t$ we may then write

$$\begin{aligned} D(\underline{p}_t + \epsilon(t))(\underline{p}_t - \epsilon(t)) &\leq q_i(p_{ij}, p_{jt}) + q_j(p_{jt}, p_{it}) \\ &\leq D(\underline{p}_t - \epsilon(t))(\underline{p}_t + \epsilon(t)) \end{aligned}$$

finally by taking limits we obtain

$$D(p^*)p^* \leq \pi_1(p^*, p^*) + \pi_2(p^*, p^*) \leq D(p^*)p^*$$

which proves the desired continuity result.

- ii) In order to prove weak lower semi-continuity for $p > P(k_1 + k_2)$ note that for any p^*

$$\begin{aligned} \liminf_{p \rightarrow p^*} \pi_i(p, p^*) &= \min(k_i, D(p^*))p^* \\ &\geq \pi_i(p^*, p^*) = \min(k_i, \frac{k_i}{k_1 + k_2} D(p^*))p^* \end{aligned}$$

□

We first consider the possibility of reaching a discounting subgame where the list prices induce a pure strategy equilibrium. A well known result of the Bertrand-Edgeworth literature is that the only candidate for a pure strategy equilibrium is the competitive price (see Arrow (1951) in Canoy). The condition under which the Arrow result applies in our model is discussed in Theorem 3. We show later that this condition is never met in equilibrium unless the classis Bertrand-Edgeworth game has a pure strategy equilibrium.

A pure strategy equilibrium that does not involve the competitive price can always be induced in the discounting stage if firm i sets its list price equal to the competitive price

(and firm j sets a list price that is high enough). In fact any pair of list prices $p_i^L > p_j^L$ such that firm i will act on the residual demand if firm j sets a list price of p_j^L ,

$$\min(D(p_j^L), k_i)p_j^L \leq \max_{p \in [p_j^L, p_i^L]} R(p, p_j^L, k_j)p$$

induce a pure strategy equilibrium in the discounting stage, $(\hat{p}_i^d, \hat{p}_j^d)$, where firm j does not discount its list price and firm i acts on the residual demand setting a discounted price of

$$\hat{p}_i^d = \arg \max_{p \in [p_j^L, p_i^L]} R(p, p_j^L, k_j)p$$

Theorem 3 Let $p_i^L \geq p_j^L$, if

$$\min(D(p_j^L), k_i)p_j^L > \max_{p \in [p_j^L, p_i^L]} R(p, p_j^L, k_j)p$$

the only candidate for a pure strategy equilibrium involves both firms setting the competitive price.

Proof: Suppose a pure strategy equilibrium to the discounting game, $(\hat{p}_1^d, \hat{p}_2^d)$, exists. If $\hat{p}_i^d < \hat{p}_j^d$, this implies that $\hat{p}_i^d = p_i^L$ or else firm i would want to raise its price, which contradicts $p_i^L \geq p_j^L$. Suppose on the other hand $\hat{p}_j^d < \hat{p}_i^d$, this implies $\hat{p}_j^d = p_j^L$. Further, in order for firm i to not have incentives to undercut firm j it must be the case that

$$\min(D(p_j^L), k_i)p_j^L \leq \max_{p \in [p_j^L, p_i^L]} R(p, p_j^L, k_j)p$$

which leads to contradiction. We have then that both firms set the same discounted price. However, if both firms set the same discounted price, it must be the case that the equilibrium is competitive (or else at least one firm will have an incentive to undercut its rival).

□

We now consider the possibility of reaching a subgame where list prices induce a non-degenerate mixed strategy equilibrium. Given the list prices set in the first stage (p_1^L, p_2^L) a firm's strategy in the discounting subgame is defined by a (possibly degenerate) probability measure μ_i^d on $[0, p_i^L]$. Let the minimum and the maximum of the support of μ_i^d be denoted by \underline{p}_i and \bar{p}_i respectively. Given any two strategies (μ_i^d, μ_j^d) a firm's expected profits in discounting stage will be denoted by $\pi_i(\mu_i^d, \mu_j^d)$.

The next result characterizes some of the properties of a non-degenerate mixed strategy equilibrium in the discounting stage. Property i) shows that the lower bound of the price support is the same for both firms and that it is above the market clearing price. Property ii) shows that if \underline{p}^d is greater than the competitive price then firms have no atoms at \underline{p}^d . One of the implications of this is that *both* firms have non degenerate mixed strategies in equilibrium. Property iii) implies that there is a firm h that when setting the highest price in its support will be undercut with certainty. It should be noted that a discounting subgame where the list prices are set at p^0 is equivalent to the classical one stage pricing game. This theorem thus generalizes some of the results of the Bertrand-Edgeworth literature which are known to hold in the case of the efficient and proportional residual demands for a more general class of residual demand functions.

Theorem 4 Given $(p_1^L > 0, p_2^L > 0)$ if a nondegenerate mixed strategy equilibrium, (μ_1^d, μ_2^d) , to the discounting subgame exists:

- i) $\underline{p}_1^d = \underline{p}_2^d = \underline{p}^d$
- ii) $\pi_i(\mu_i^d, \mu_j^d) = \min(D(\underline{p}^d), k_i)\underline{p}^d$ for any $i \in \{1, 2\}$
- iii) for one of the two firms $h \in \{1, 2\}$:

$$\pi_h(\mu_h^d, \mu_{-h}^d) = \int_{\underline{p}^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h})\bar{p}_h^d d\mu_{-h}^d(p)$$

Proof: See Appendix

□

As a corollary to Theorem 1 we will prove that the lower bound of the support of the mixed strategy equilibrium is below the Edgeworth price of firm 1. This result is important since it implies that firm 1 would be better off if it could commit to a price of p_1^E and have firm 2 act on the residual demand than in any discounting game that has a non-degenerate mixed strategy equilibrium. The proof is based on the fact that by Theorem 2 in a mixed strategy equilibrium there is a firm h which sets a price of \bar{p}_h^d and is undercut by its rival with certainty. Firm h 's payoffs when setting this price are not certain, they are greatest when its rival sets a price of \underline{p}^d , thus expected profits of firm h are strictly less than $R(\bar{p}_h^d, \underline{p}^d, k_{-h})\bar{p}_h^d$. Thus if firm $-h$ were to set a price sufficiently close to \underline{p}^d with certainty, firm h would best respond acting on the residual demand. This in turn implies the Edgeworth price of firm h must be greater than \underline{p}^d .

Corollary 1 Given $(p_1^L > 0, p_2^L > 0)$ if a non-degenerate mixed strategy equilibrium to the discounting game exists then $\underline{p}^d < p_1^E$

Proof: By Theorem 4 there is a firm h for which

$$\min(D(\underline{p}^d), k_h) \underline{p}^d = \int_{\underline{p}^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h}) \bar{p}_h^d d\mu_{-h}(p)$$

given the continuity of the residual demand function by the Mean Value Theorem we have that

$$\int_{\underline{p}^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h}) \bar{p}_h^d d\mu_{-h}(p) = R(\bar{p}_h^d, z, k_{-h}) \bar{p}_h^d$$

for some $\underline{p}^d < z < \bar{p}_{-h}^d$. By Property (5) of residual demand we have

$$\min(D(\underline{p}^d), k_h) \underline{p}^d < R(\bar{p}_h^d, \underline{p}^d, k_{-h}) \bar{p}_h^d$$

which along with $p_i^E \leq p_1^E$ gives us the desired result.

□

4.2. The Full Game

We now characterize the subgame perfect equilibrium of the list pricing game. Our first result is that if the Edgeworth price of both firms coincides with competitive price then any subgame perfect equilibrium of the list pricing game involves both firms setting the competitive price.

Theorem 5 If $p_1^E = P(k_1 + k_2)$, any subgame perfect equilibrium of the list pricing game involves both firms setting a discounted price of $P(k_1 + k_2)$.

Proof: Note that a firm i can always guarantee itself profits of $P(k_1 + k_2)k_i$ by setting the competitive price in the list pricing stage and in the discounting stage, this implies that any list price below the competitive price is strictly dominated. For this reason in a subgame perfect equilibrium no firm will set a price below $P(k_1 + k_2)$.

We now prove that if a firm j sets a price of $P(k_1 + k_2)$ in the discounting stage the best response of firm i is to set a price of $P(k_1 + k_2)$. Given that $p_j^E = P(k_1 + k_2)$ we have that

$$k_i P(k_1 + k_2) = \max_{p \in [P(k_1 + k_2), p^0]} R(p, P(k_1 + k_2), k_j) p$$

Furthermore by property (4) of the residual demand

$$k_i P(k_1 + k_2) = R(P(k_1 + k_2), P(k_1 + k_2), k_j) P(k_1 + k_2)$$

This along with property (3) yields

$$k_i P(k_1 + k_2) > R(p, P(k_1 + k_2), k_j) p$$

for any $p > P(k_1 + k_2)$, which proves the desired result.

This proves that setting the competitive price is an equilibrium. To prove uniqueness note that if $p_i^L = P(k_1 + k_2)$ for some firm i , then firm i will set the competitive price in the discounting stage and by the previous argument firm j will best respond by setting the competitive price. If on the other hand $p_i^L > P(k_1 + k_2)$ for both firms by Corollary 1 no nondegenerate mixed strategy equilibrium will exist.

□

The intuition behind the result is that if both firms set the competitive price no firm has an incentive to raise its price since the characterization of the Edgeworth price implies $P(k_1 + k_2)k_i = \max_{p' \in [p_j, p_0]} R(p', P(k_1 + k_2), k_j)p$. To see that this is the only possible equilibrium note that by Corollary 1 when $p_1^E = P(k_1 + k_2)$ and $p_i^L \geq P(k_1 + k_2)$, for some i , a mixed strategy equilibrium to the discounting subgame does not exist.

Since the Bertrand-Edgeworth pricing game can be seen as a discounting game where the list prices are set arbitrarily high it is clear that by Theorem 5 if $p_1^E = P(k_1 + k_2)$ the Bertrand-Edgeworth model has a pure strategy equilibrium. On the other hand by Theorem 3 we have that the only candidate for a pure strategy equilibrium in a Bertrand-Edgeworth model is the competitive price, but if $p_1^E > P(k_1 + k_2)$ firm 2 will have an incentive to deviate from this equilibrium. This leads to our next result which characterizes when the one stage pricing game has a pure strategy equilibrium.

Corollary 2 The Bertrand-Edgeworth model has a pure strategy equilibrium iff $p_1^E = p_2^E = P(k_1 + k_2)$

Theorem 5 and Corollary 2 imply that when the Bertrand-Edgeworth model has a pure strategy equilibrium the addition of a list pricing stage is innocuous in the sense that it leads to the same prices in equilibrium. We will now deal with the case of characterizing the equilibria of the list pricing game when a pure strategy equilibrium of the Bertrand-Edgeworth game does not exist.

In the following theorem we prove the existence of a subgame perfect equilibrium of the list pricing game in which firms play pure strategies. In this subgame perfect equilibrium, which we denote by e^* , firm 1 sets its list price equal to p_1^E and does not discount, while firm 2 sets its list price arbitrarily high and acts as a monopolist on the residual demand in the discounting stage.

Theorem 6 $(p_1^L = p_1^d = p_1^E, p_2^L \geq p^M(p_1^E, k_1) = p_2^d)$ is a subgame perfect equilibrium of the list pricing game.

Proof: If $p_2^E = P(k_1 + k_2)$ this implies that $p_1^E = P(k_1 + k_2)$ and we obtain the desired result by applying Theorem 5. Suppose on the other hand that $p_2^E > P(k_1 + k_2)$. In this case the proposed equilibrium yields the following profits:

$$\begin{aligned}\pi_1^* &= \min(k_2, D(p_1^E))p_1^E \\ \pi_2^* &= R(p^R(p_1^E, k_1), p_1^E, k_1)p^R(p_1^E, k_1) = \min(k_2, D(p_1^E))p_1^E\end{aligned}$$

Let us suppose firm i has a profitable deviation. It must involve setting a list price greater than p_i^E . Corollary 2 implies that there is no pure strategy equilibrium. Given that an equilibrium exists it must be a mixed strategy equilibrium (μ_i, μ_j) . Let us denote the expected profits in this equilibrium by (π_i, π_j) . From Theorem 4 we have that for some firm $h \in \{1, 2\}$

$$\pi_h = \int_{\underline{p}^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h})\bar{p}_h^d d\mu_{-h}(p)$$

Applying the mean value theorem and given the fact that the equilibrium is nondegenerate we obtain that for some $p' \in (\underline{p}^d, \bar{p}_h^d)$,

$$\pi_h = R(\bar{p}_h^d, p', k_{-h})\bar{p}_h^d$$

Thus by property (5) of residual demand

$$\pi_h < R(\bar{p}_h^d, \underline{p}^d, k_{-h})\bar{p}_h^d$$

this in turn implies that $\underline{p}^d < p_h^E$ and therefore $p^{MIN} < p_h^E < p_1^E$. This along with the fact that $\pi_i = \min(D(p^{MIN}), k_i)p^{MIN}$ proves that $\pi_i^* > \pi_i$.

□

We now prove that any subgame perfect equilibrium that does not lead to the same price outcome as e^* leads to lower expected payoffs for both firms. This property makes it a clear focal point of the list pricing game e^* , furthermore if there is any pre-play communication firms would coordinate to this equilibrium.

Theorem 7 Any subgame perfect equilibrium of the list pricing game that does not result in prices of p_1^E and $p^M(p_1^E, k_1)$ leads to lower expected profits for both firms.

Proof: If $p_1^E = P(k_1 + k_2)$ then from Theorem 5 we have that e^* is the unique equilibrium.

Suppose on the other hand that $p_1^E > P(k_1 + k_2)$ and that there exists an equilibrium \hat{e} that leads to an expected payoff of $\hat{\pi}_i$, where $\hat{\pi}_i > \pi_i^*$ for at least one $i \in \{1, 2\}$. We will denote the lower bound of the list price support of a firm i in \hat{e} by \underline{p}_i^L . We first note that if this equilibrium is not to be dominated for both firms it must be the case that $\underline{p}_i^L \geq p_1^E$ for some firm i . Without loss of generality we assume that $\underline{p}_i^L \geq \underline{p}_j^L$. We will characterize the expected payoff of \hat{e} by the expected payoff of a discounting game where the list prices are given by $(p_i^L, \underline{p}_j^L)$, for any $p_i^L \geq \underline{p}_j^L$. It cannot be the case that

$$\min(D(\underline{p}_j^L), k_i) \underline{p}_j^L < \max_{p \in [\underline{p}_j^L, \underline{p}_i^L]} R(p, \underline{p}_j^L, k_j) p$$

since strategy \underline{p}_j^L is dominated by $\underline{p}_j^L + \epsilon$ for any $\epsilon > 0$ for which

$$\min(D(\underline{p}_j^L + \epsilon), k_i) \underline{p}_j^L + \epsilon < \max_{p \in [\underline{p}_j^L, \underline{p}_i^L]} R(p, \underline{p}_j^L, k_j) p$$

It must then be the case that

$$\min(D(\underline{p}_j^L), k_i) \underline{p}_j^L \geq \max_{p \in [\underline{p}_j^L, \underline{p}_i^L]} R(p, \underline{p}_j^L, k_j) p$$

For any pair $(p_i^L, \underline{p}_j^L)$ for which

$$\min(D(\underline{p}_j^L), k_i) \underline{p}_j^L > \max_{p \in [\underline{p}_j^L, \underline{p}_i^L]} R(p, \underline{p}_j^L, k_j) p$$

the discounting subgame will either have a nondegenerate mixed strategy equilibrium and by Corollary 1 $\hat{\pi}_i > \pi_i^*$ or a pure strategy equilibrium and by Theorem 5 will result in competitive prices, thus $\hat{\pi}_i > \pi_i^*$. It must be the case that for all but a subset of measure zero of the list price pairs played $(p_i^L, \underline{p}_j^L)$,

$$\min(D(\underline{p}_j^L), k_i) \underline{p}_j^L = \max_{p \in [\underline{p}_j^L, \underline{p}_i^L]} R(p, \underline{p}_j^L, k_j) p$$

Note that for this inequality to hold it must be the case that $\underline{p}_j^L \leq p_1^E$. If $\underline{p}_j^L < p_1^E$ then $\hat{\pi}_i > \pi_i^*$. Thus it must be the case that $\underline{p}_j^L = p_1^E$ and thus $p_i^L \geq p^M(p_1^E, k_j)$ which implies that all but a measure zero of the price outcomes in \hat{e} are given by $p_1^d = p_1^E$ and $p_2^d = p^M(p_1^E, k_1)$.

□

A similar equilibrium outcome as in e^* is obtained in Gelman and Salop(1983). They analyze a game of entry in a market by a capacity constrained firm, where the entrant must commit to a price to which the incumbent best responds. Note, Gelman and Salop (1983) refer to the entrants low price (small size) strategy as “judo economics”. In our model the low pricing strategy is followed by the firm with the highest Edgeworth price, without any additional assumptions on the residual demand function it is not straightforward to prove a direct relation between the low price strategy and size. If we assume that residual demand is given by $R_\lambda(p_i, p_j, k_j)$ for any $\lambda < 1$ then it is straightforward to prove that $k_i < k_j$ implies $p_i^E > p_j^E$, that is the small firm will follow the low pricing strategy. There are several examples of the empirical validity of this type of “judo economics” pricing behavior which are given in Gelman and Salop (1983), and Sorgard (1995).

5. List Pricing and Price Leadership

Price leadership has been studied in the literature with endogenous determination of the timing of the moves, i.e., whether a firm prefers to act as a leader, or as a follower. In these models, once a firm sets its price it cannot be changed regardless of how the rival responds. Even though ex-post it would be in the leader’s interest to change its price none of the papers explain the strong nature of this commitment. In this section we argue that list pricing may provide such a credible commitment mechanism in which price outcomes emerge that are *similar* to price leadership.

Hamilton and Slutsky (1990) propose a two stage framework to endogenize the timing of a duopoly game where each firm chooses a strategy (which could be price or quantity). Firms may choose their strategy in period 1 or wait till period 2. If a firm chooses a strategy in the first period and the other firm waits it is informed of the strategy chosen by its rival. In another paper van Damme and Hurkens (1996) show that playing simultaneously is subgame perfect in the Hamilton-Slutsky timing game only if none of the players has an incentive to move first.

We will now show that the sub-game perfect equilibria of our list pricing game is a sub-game perfect equilibria in the endogenous timing framework proposed by Hamilton and Slutsky. In order to obtain our equivalence result it suffices to prove that in the Bertrand-Edgeworth game that we analyze the mixed strategy equilibria is indeed dominated by a sequential game where firm 2 moves first.

Theorem 8 When no pure strategy equilibrium of the Bertrand-Edgeworth game exists

firm 2 has an incentive to move first.

Proof: By Theorem 4 and Corollary 1 in any mixed strategy equilibrium of the discounting game (including the case where list prices are set arbitrarily high) the expected payoff of firm 2 is given by $\underline{p}^d \min(D(\underline{p}^d), k_2)$ for some $\underline{p}^d < p_2^E$.

On the other hand if firm 2 moves first and sets a price of p_2^E then firm 1 will set $p^M(p_2^E, k_2)$ and firm 2 will obtain profits of $p_2^E \min(D(p_2^E), k_2)$.

□

We then have that the subgame perfect equilibria involves one firm moving first. We will now prove that in Hamilton and Slutsky timing game if firm i moves first it will set its Edgeworth price.

Theorem 9 When no pure strategy equilibrium of the Bertrand-Edgeworth game exists if firm i moves first it will set a price of p_i^E .

Proof: Suppose firm i is moving first. If it sets a price above p_i^E it will be undercut by firm j in the second stage and its profits will be bounded by $\hat{\pi}_i = (D(\hat{p}) - k_j)\hat{p}$ where

$$\hat{p} = \arg \max_p (D(p) - k_j)p$$

By Theorem 4 if firm i deviates to simultaneous play in the second stage it will obtain expected profits of $\pi_i^s = \underline{p} \min(D(\underline{p}), k_i)$. We will now prove that $\hat{\pi}_i < \pi_i^s$. Suppose on the other hand $\hat{\pi}_i \geq \pi_i^s$, it must be the case that $\hat{p} > \underline{p}$. Suppose that firm i deviates from its mixed strategy and sets a price of \hat{p} . Given that the equilibrium is nondegenerate $\mu_j([p, \hat{p})) > 0$. The payoffs of firm i from this deviation will be bounded below by $R(\hat{p}, p', k_j)\hat{p}$ where $p' \in [p, \hat{p})$. By properties 4 and 5 of the residual demand function

$$R(\hat{p}, p', k_j)\hat{p} > (D(\hat{p}) - k_j)\hat{p}$$

thus $\hat{\pi}_i < \pi_i^s$. From this we may conclude that if firm i moves first it will choose a price less than or equal to p_i^E . Given that for firm i any price below p_i^E is dominated by p_i^E we obtain the desired result.

□

Finally, it is straightforward to see that given $p_2^E > p_1^E$ if there exists a subgame perfect equilibrium where firm 1 leads it is dominated for both firms by the equilibrium where firm 2 leads.⁸ We have then proved that in our model the sequential-timing and list-pricing solutions to the nonexistence of a pure strategy equilibria are equivalent. The difference is that while in the sequential-timing models firms are not allowed to change their price (once it is chosen), in our list pricing approach firms can discount. Our result is obtained under a weaker assumption that reflects a pricing institution that is prevalent in many markets.

6. Conclusion

The mixed strategy equilibrium result in Bertrand-Edgeworth models has been criticized as an unsatisfactory explanation of firm pricing behavior in oligopolistic markets. Several authors have addressed the non-existence issue. When the number of firms in the industry is arbitrarily large Allen and Hellwig (1986a), Vives (1986), Borgers (1986) and Dixon (1986) show that the mixed strategy equilibrium outcome approximates the pure strategy competitive equilibrium. Hence, as pointed out by Allen and Hellwig (1986b) the nonexistence issue is “in some sense unimportant”.

For the duopoly case Shubik and Levitan (1980), DK, and Canoy (1996) avoid the non-existence problem by imposing a sequential timing structure in firm pricing moves. Thus one of the two firms must commit to a price which cannot be changed when its rival best responds. Given that the price leader will ex-ante have an incentive to change its price commitment has to be credible. In our paper adding list pricing into the standard Bertrand-Edgeworth model makes commitment credible and we obtain a pure strategy outcome.

As in DK, in our paper also capacities determine the leader. However, *unlike* DK the small firm emerges as the leader in our structure. Further, in DK Stackelberg leadership prices outcomes are more collusive than the simultaneous move setting. We get a similar result, however, our result arises from a simultaneous move two period list pricing game without discounting. Finally, as in DK in our paper we have a deterministic price solution.

In our equilibrium one of the firms commits to a low price signalling to its rival that it can act as a monopolist on the residual demand. Our result suggests that the traditional one-stage pricing Bertrand-Edgeworth models may overstate the competitiveness of an oligopolistic industry (DK make a similar point). Credible commitment to price by a firm

⁸It should be noted that when residual demand takes on the form of R_λ our results imply that the smallest firm will adopt a “leadership” role.

can enforce a pure strategy outcome. Further, an interesting result arising from our paper is that in many cases it is in the interest of the small firm to commit credibly and choose its price first. In this sense, unlike the general interpretation, the “leadership” role (i.e. first mover) is assumed by the smaller firm.

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7. Appendix

The following two Lemmas will prove useful in order to prove Theorem 4

Lemma 1. *In a mixed strategy equilibrium to the discounting subgame if $\mu_i([\hat{p}, \hat{p}+\epsilon)) > 0$ for any $\epsilon > 0$ and $\pi_i(p)$ is right continuous at $p = \hat{p}$ then the expected profit of the equilibrium is given by $\pi_i(\hat{p})$.*

Proof: i) If $\mu_i(\hat{p}) > 0$ the proof is trivial, consider the case of $\mu_i(\hat{p}) = 0$ and suppose $\pi_i(\hat{p}) - \pi_i^* = C$ where $C < 0$ and π_i^* is the expected payoff of the mixed strategy equilibrium. Since $\mu_i((\hat{p}, \hat{p}+\epsilon)) > 0$ for any $\epsilon > 0$ it must be the case that $\pi_i(\hat{p}) = \pi_i^*$ for some $p \in (\hat{p}, \hat{p} + \epsilon)$.

On the other hand by right continuity of $\pi_i(p)$ at $p = \hat{p}$ we have then that for any $\delta > 0$, there exists an $\epsilon > 0$ s.t. $0 < \hat{p} - p < \epsilon$ implies $|\pi_i(\hat{p}) - \pi_i(p)| < \delta$. Take $\delta = \frac{C}{2}$ and we reach a contradiction with $|\pi_i(p) - \pi_i^*| = C$.

□

Lemma 2. *In a mixed strategy equilibrium to the discounting subgame if firm i has positive measure at a price \hat{p} , $P(k_1 + k_2) < \hat{p} \leq p_i^L$ then $\mu_j([\hat{p}, \hat{p} + \epsilon]) = 0$ for small enough $\epsilon > 0$.*

Proof: For any $p < \hat{p}$ we may write the expected profits expression as

$$\pi_j(p) = (\mu_i((p, p)) + \mu_i([p, p^R])) \min(k_j, D(p))p + \int_{[0, \hat{p}] - [p, \hat{p}]} R(p, z, k_i) d\mu_i(z) + \mu_i(p) D(p) p \frac{k_j}{k_1 + k_2}$$

Taking limits from the left

$$\lim_{p \rightarrow \hat{p}} \pi_j(p) = \mu_i([\hat{p}, p^0]) \min(k_j, D(\hat{p})) \hat{p} + \int_{[0, \hat{p})} R(\hat{p}, z, k_i) d\mu_i(z)$$

Consider now the expected profit function when $p > \hat{p}$

$$\pi_j(p) = (\mu_i([p, p^0]) - \mu_i([\hat{p}, p])) \min(k_j, D(p))p + \int_{[0, \hat{p}] \cup (\hat{p}, p)} R(p, z, k_i) d\mu_i(z) + \mu_i(p) D(p) p \frac{k_j}{k_1 + k_2}$$

Taking limits from the right

$$\lim_{p \rightarrow +p} \pi_j(p) = \mu_i((\hat{p}, p^0]) \min(k_{-i}, D(p))p + \int_{[0, \hat{p}]} R(\hat{p}, z, k_i) d\mu_i(z)$$

On the other hand we have that

$$\pi_j(\hat{p}) = \mu_i((\hat{p}, p^0]) \min(k_{-i}, D(\hat{p}))\hat{p} + \int_{[0, \hat{p}]} R(\hat{p}, z, k_i) d\mu_i(z) + \mu_i(\hat{p}) D(\hat{p}) \hat{p} \frac{k_j}{k_1 + k_2}$$

We have then

$$\pi_j(\hat{p}) - \lim_{p \rightarrow \hat{p}} \pi_j(p) = \mu_i(\hat{p}) (D(\hat{p}) \frac{k_j}{k_1 + k_2} - \min(k_j, D(\hat{p}))) \hat{p}$$

Note that since $\hat{p} > P(k_1 + k_2)$ then $D(\hat{p}) \frac{k_j}{k_1 + k_2} < \min(k_j, D(\hat{p}))$. Thus $\pi_j(\hat{p}) - \lim_{p \rightarrow \hat{p}} \pi_j(p) < 0$ which implies there exists a $p < \hat{p}$ that gives firm j higher expected profits than \hat{p} thus $\mu_j(\hat{p}) = 0$.

On the other hand we have

$$\pi_j(\hat{p}) - \lim_{p \rightarrow +\hat{p}} \pi_j(p) = \mu_j(\hat{p}) (D(\hat{p}) \frac{k_j}{k_1 + k_2} - R(\hat{p}, \hat{p}, k_i)) \hat{p}$$

Note that since $p > P(k_1 + k_2)$ and by property (4) of the residual demand function $D(\hat{p}) \frac{k_j}{k_1 + k_2} - R(\hat{p}, \hat{p}, k_i) > 0$. Thus $\pi_j(\hat{p}) - \lim_{p \rightarrow +\hat{p}} \pi_j(p) > 0$ which implies that there exists an $\epsilon > 0$ such that $\pi_{-i}(p) < \pi_{-i}(\hat{p})$ for $p \in (\hat{p}, \hat{p} + \epsilon]$. This implies $\mu_j((\hat{p}, \hat{p} + \epsilon]) = 0$.

□

Proof: [Proof of Theorem 4-i] Suppose firms have different lower bounds for their support, thus $\underline{p}_i^d < \underline{p}_j^d$.

i) Consider the case $\underline{p}_j^D \leq p_i^M$ (note that this implies $p_i^M > 0$ since $\underline{p}_i^d \geq 0$), we have then that pure strategy $\lambda \underline{p}_i^d + (1 - \lambda) \underline{p}_j^d$ for any $0 < \lambda < 1$ dominates any strategy in $[\underline{p}_i^d, \lambda \underline{p}_i^d + (1 - \lambda) \underline{p}_j^d]$. On the other hand by definition of support we have that $\mu_i([\underline{p}_i^d, \underline{p}_i^d + \epsilon]) > 0$ for any $\epsilon > 0$. This implies that $\underline{p}^d = p_i^L$ and $\mu_i(p_i^L) = 1$. It must then be the case that

$$\min(k_i, D(p_i^L)) p_i^L \leq \max_{p' \in [p_i^L, p_j^L]} R(p', p_i^L, k_i) p'$$

and thus firm j maximizes its profits by playing $\arg \max_{p' \in [p_i^L, p_j^L]} R(p', p_i^L, k_i)p'$ this contradicts the assumption that firms are playing nondegenerate mixed strategies in equilibrium.

ii) Suppose $\underline{p}_j^d > p_i^M$ then p_i^M dominates any other price for firm i . Thus the best response by firm i involves playing a pure strategy $p_i^d = \min(p_i^M, p_i^L)$. If $p_i^d = p_i^L$ by the previous argument firm j must be playing the pure strategy $\arg \max_{p' \in [p_i^L, p_j^L]} R(p', p_i^L, k_i)p'$. If on the other hand $p_i^d = p_i^M$ firm j faces a residual demand of zero and the best response of firm j must involve undercutting p_i^d which contradicts $\underline{p}_i^d < \underline{p}_j^d$.

□

Proof: [Proof of Theorem 4-ii] By definition of support it must be the case that $\mu_i([\underline{p}^d, \underline{p}^d + \epsilon)) > 0$ for any $\epsilon > 0$. We will now prove right continuity of the firms expected profit function at \underline{p}^d and apply Lemma 1 to obtain the desired result.

Let us consider the expected profits for a particular firm i for a particular strategy $p > \underline{p}^d$

$$\begin{aligned} \pi_i(p) = & \mu_j((p, p^0)) \min(k_i, D(p))p + \int_{\{\underline{p}^D\} \cup (\underline{p}^D, p)} R(p, z, k_j)pd\mu_j(z) + \\ & \mu_j(p)D(p)p \frac{k_i}{k_1 + k_2} \end{aligned}$$

Taking the limit from the right

$$\begin{aligned} \lim_{p \rightarrow \underline{p}^D} \pi_i(p) = & \mu_j((\underline{p}^d, p^0)) \min(k_i, D(\underline{p}^d))\underline{p}^d + \int_{\underline{p}^D} R(\underline{p}^d, z, k_j)\underline{p}^d d\mu_j(z) = \\ & \mu_j((\underline{p}^d, p^0)) \min(k_i, D(\underline{p}^d))\underline{p}^d + \mu_j(\underline{p}^d)R(\underline{p}^d, \underline{p}^d, k_{-i})\underline{p}^d \end{aligned}$$

Suppose now that $\underline{p}^d = P(k_1 + k_2)$, then $R(\underline{p}^d, \underline{p}^d, k_j)\underline{p}^d = k_i P(k_1 + k_2)$, and $\lim_{p \rightarrow P(k_1 + k_2)} \pi_i(p) = k_i P(k_1 + k_2)$. Note that $\pi_i(P(k_1 + k_2)) = k_i P(k_1 + k_2)$. We have then proven right continuity of the expected profit function of firm i when $\underline{p}^d = P(k_1 + k_2)$.

Suppose on the other hand $\underline{p}^d > P(k_1 + k_2)$. We will first note that $\mu_i(\underline{p}^d) = 0$ for $i \in \{1, 2\}$, if this were not the case for some i , then by Lemma 2 we would have $\mu_j([\underline{p}^d, \underline{p}^d + \epsilon)) = 0$ for some $\epsilon > 0$, which contradicts the fact that \underline{p}^d is in the support of firm j . We have then

$$\lim_{p \rightarrow \underline{p}^D} \pi_i(p) = \mu_{-i}((\underline{p}^d, p^0)) \min(k_i, D(\underline{p}^d))\underline{p}^d = \min(k_i, D(\underline{p}^d))\underline{p}^d$$

□

Proof: [Proof of Theorem 4-iii] Let us define $\bar{p} = \max(\bar{p}_1, \bar{p}_2)$. Suppose $\mu_i(\bar{p}) > 0$ for some i , by Lemma 2 $\mu_j(\bar{p}) = 0$ and thus $h = i$. Suppose $\mu_i(\bar{p}) = 0$ for both i . It must be the case that $\bar{p} = \bar{p}_i$ for some i , we then have $h = i$.

□